Design Tables for an Elliptic-Function Band-Stop Filter ($N=5$)

B. M. SCHIFFMAN, MEMBER, IEEE, AND L. YOUNG, SENIOR MEMBER, IEEE

Abstract—**Design tables for a wide-band elliptic-function band-stop transmission-line filter are described. The filter consists of five alternating quarter-wave and half-wave stubs separated by quarter-wave lines. The characteristic impedances of the filter elements were computed from published tables of the lumped-element prototype by an exact method. The filter elements are assumed to be lossless.**

Introduction

THE EXACT design method for microwave filters constructed of commensurate lengths of transmission line (one quarter-wave long at the design frequency) is at present well established [1]–[7]. In essence, this method consists of starting with a lumped-element (LE) filter and then replacing each lumped element by its transmission-line counterpart. The response of the transmission-line (TL) filter is then exactly known, and is related to its LE counterpart according to the following rules. The response of the LE filter over the entire frequency range, from zero to infinite frequency, is mapped into a finite portion of the TL filter domain, from zero to a certain design frequency, and this response is repeated endlessly at progressively higher frequencies in the TL filter domain. Thus, considering only the first pass band and stop band, low-pass LE filters are transformed into band-stop TL filters, and high-pass LE filters are transformed into band-pass TL filters. A practical configuration of the resulting stub-type filter is then obtained with the aid of Kuroda's identities, to be described later.

The availability of design tables for lumped-element filters having maximally flat or Chevyshev responses paved the way for the computation of corresponding tables for transmission-line filters. Such tables for band-stop filters have been published by the Stanford Research Institute [5]. Each filter consists of open-circuit shunt stubs separated by lengths of transmission line, and the stop band of each filter is centered at the design frequency. In this paper we describe tables for a band-stop filter based on an elliptic-function LE low-pass filter prototype [8], [9].

Recent developments in circuit theory by R. Levy [10][1] suggested the computer synthesis of these band-stop filter designs from tables of the elliptic-function low-pass filter. (The tables are published by Telefunken G.M.B.H. of Western Germany [9].) Both the Kuroda and the Kuroda-Levy identities are used in this synthesis procedure. The prototype filter has resonant L-C sections and is therefore slightly more complex in configuration (although much more complex from the viewpoint of circuit theory) than the simple low-pass ladder structure; hence the band-stop filter derived therefrom is more complex than the maximally flat or Chebyshev type of filter. This band-stop filter has both half-wave and quarter-wave stubs, the half-wave stubs providing the frequencies of infinite attenuation in the stop band at points other than the center frequency. The minimum value of the attenuation ripples in the stop band is an additional design parameter that is not present in the simpler filter types.

The tables are limited to filter designs having only five stubs ($N=5$). There are several reasons for this limitation. First, the additional design parameter mentioned above resulted in greatly enlarged tables. Second, the results for the $N=5$ case included many filter designs having stubs of very high characteristic impedance, and it was therefore concluded that very few practical designs would result for still higher values of N. This conclusion was based on the fact that each application of the Kuroda and the Kuroda-Levy identities in the synthesis procedure leads to higher values of impedance, and the number of times these identities are used increases with N. Finally, lower values of N ($N=3$ is the lowest value for the elliptic-function filter) were not at first considered for computer synthesis because the Kuroda-Levy formulas are not required for $N=4$ and $N=3$, and band-stop filters for these two cases could easily be designed from previously known methods. The Kuroda-Levy formulas are not required for these cases because for $N=3$ or 4 there is only one half-wave stub, and unit elements can be worked into the filter from each end by means of the relatively easy-to-use Kuroda's identities.[2] In these cases the half-wave stub remains unchanged.

Use of the Tables

To use the design tables, it is only necessary to understand the symbols therein and how they relate to the circuit diagram of the band-stop transmission-line filter

Manuscript received April 8, 1966; revised June 29, 1966. This work was sponsored by the Electronic Components Laboratory of the U. S. Army Electronics Command, Fort Monmouth, N. J., under Contract DA 28-043 AMC-01271(E).

The authors are with the Electromagnetic Techniques Laboratory, Stanford Research Institute, Menlo Park, Calif.

[1] Levy's work (see [10]) contained a transformation of circuit elements required in the practical realization of a distributed-element elliptic-function filter. Upon the completion of our research we discovered that the same transformation was first proposed by Professor K. Kuroda (see [15]). In this paper the transformation alluded to, which will be described later, will be called the Kuroda-Levy identity.

[2] See, for example, B. M. Schiffman, "A harmonic rejection filter designed by an exact method," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 58–60, January 1964, and "A multi-harmonic rejection filter designed by an exact method," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 512–516, 1964.

of Fig. 1, and to the filter response sketched in Fig. 2. These symbols are fully defined in Table I and are further explained below. A portion of the design tables is reproduced in Table II. There are 33 pages in all.[3]

The filter consists of three quarter-wave and two half-wave stubs in alternating sequence, with adjacent stubs separated by a quarter-wavelength of line. The normalized characteristic impedance Z_i or $Z_{i,i+1}$ of each quarter-wave section of the filter is given in the design tables. (The normalizing impedances are the equal filter terminating impedances.) These TL sections may be constructed in coaxial line or strip line. (The circuit of Fig. 1 shows two-wire line.) Each horizontal row in the tables is one complete design. The parameters from which the filter designer can choose are the maximum absolute value P of the reflection coefficient in the pass band, the minimum attention $A(S)$ in the stop band, and the width of the stop band $W(P)$ at the pass band equi-ripple points. The designer may also substitute $W(S)$, the width of the stop band as shown in Fig. 2 as a design parameter in place of $W(P)$.

The value of $P(0<P<1)$ is shown in the upper right-hand heading of each subgroup of the design tables. The maximum voltage standing-wave ratio in the pass band is related to P by the formula

$$\text{VSWR(max)} = \frac{1+P}{1-P}\,. \tag{1}$$

The values of P given in the tables are $P=0.01$, 0.03, 0.05, 0.10, 0.15, 0.25, and 0.50, representing a range of pass-band maximum VSWR's of 1.02 at the beginning, to 3.0 at the end of the tables.

The various designs are given in groups, arranged according to values of P and $A(S)$. The value of $A(S)$ in decibels for each design is shown on the second line of the heading of each subgroup. These $A(S)$ values range from approximately 100 dB down to 20 dB, and this range is repeated for each value of P. Also shown on the second line of each heading are parameters θ, $\Omega(S)$, and $\Omega(1)$ relating to the LE prototype. On the third line of the heading of each subgroup of the tables are the element values of the LE elliptic-function filter. These values, as well as the previous three parameters, are only given for reference, and are not needed in the design of a band-stop filter. The column headings of the tables are $W(P)$, $W(S)$, and the impedances Z_i, or $Z_{i,i+1}$ of the quarter-wave sections. (The prime symbol is indicated by an asterisk.) The latter are given in the same order as they are shown in Fig. 1. It will be noted, with reference to Fig. 2, that the bandwidths $W(S)$ and $W(P)$ are normalized bandwidths, with the center frequency of the stop band (the design frequency) used as the normalization frequency. Thus, the range of $W(P)$ is $0<W(P)<2$, while $W(S)<W(P)$, always.

The design tables do not include either very small or very large values of the normalized bandwidth $W(P)$, and some subgroups representing the low values of $A(S)$ have few $W(P)$ entries. The small values of normalized bandwidth $W(P)$ approaching zero in each case, have been omitted because they resulted in excessively high impedance values of the end section of at least one of the half-wave filter stubs. On the other hand, the wider bandwidths tended to cause at least one of the quarter-wave stubs to have a negative impedance and were omitted for that reason.[4]

Once a desired filter response is specified, the tables can be examined to see if they contain one or more designs that yield a response as good as or better than specified. The range of parameters available in the design tables is given in short form in Table III, which lists the maximum and minimum values of the parameter $A(S)$ for each value of the reflection coefficient P, as well as the ranges of $W(P)$ and $W(S)$ for each given value of $A(S)$. Thus, Table III is a useful aid in quickly determining whether or not a solution for a particular specification is listed in the tables. For borderline solutions, however, the design tables must be examined directly. If no solution is found in Table III, the designer may relax the specifications or else consider a Chebyshev-type filter with $N\geq 7$ [5]. In the latter case, all stubs would be a quarter-wave long.

As previously pointed out, a large portion of all the listed designs contains very high values of normalized impedances. This means that where a stub has too high an impedance to realize per se, approximate realization methods [11], [12] may be used.[4] (Here again, a Chebyshev-type filter may be preferred with $N\geq 7$.) The design will then no longer be exact, and the filter designer will have to accept close to theoretical performance over some restricted bandwidth, with poorer performance in other portions of the band (usually the higher-frequency range). One may, for example, substitute a capacitively-coupled short-circuited stub of medium impedance, which is not quite one-quarter wave long at the design frequency, for a high-impedance exact-design stub. One way to do this is to make the reactance and the slope parameter of the substitute stub exactly match that of the original stub at the design frequency (center of the stop band). The filter will then perform very well in that region and will generally be satisfactory at lower frequencies. The technique for making this substitution has been worked out and has yielded good results in the past [11], [12]. Another way to design the substitute capacitively-coupled stub is to make the reactance of the substitute and the exact stub

[3] Deposited with ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington 25, D. C. To obtain a copy: Document 8983; advance payment $5.00 photoprints, $2.25 35mm microfilm, payable to Chief, Photoduplication Service, Library of Congress.

[4] R. Levy and I. Whitely, "Synthesis of elliptic-function filters from lumped-constant prototypes," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-14, November 1966, to be published. Here the authors showed that the exact design method is not subject to either practical or theoretical bandwidth limitations if the stub resonators are replaced by coupled-line resonators of the appropriate form.

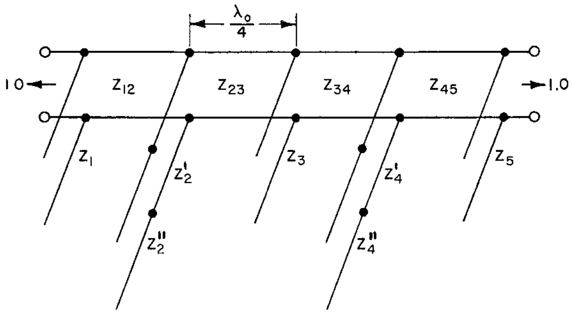


Fig. 1. Transmission-line band-stop filter with capabilities for an equi-ripple response in both pass band and stop band.

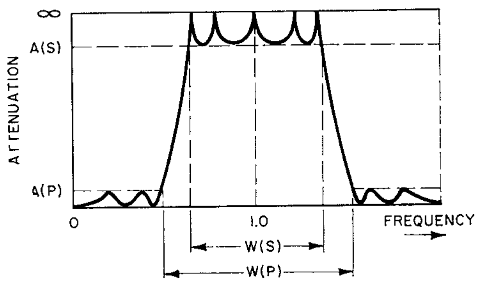


Fig. 2. Response characteristic of the filter of Fig. 1 designed according to the tables.

TABLE I

EXPLANATION OF SYMBOLS IN TABLE II AND FIGS. 1 AND 2

Symbol	Explanation
N	Total number of stubs in the filter
P	Maximum value of reflection coefficient in the pass band
Theta	Modular angle of elliptic-function (prototype) filter; an index in the table of lumped-element filters; $\theta = \arcsin 1/\Omega(S)$
$A(S)$	Minimum attenuation value of ripples in the stop band
Omega (1) or $\Omega(1)$	Prototype cutoff frequency (cps)
Omega (S) or $\Omega(S)$	Prototype skirt frequency yielding stop-band ripple attenuation $A(S)$
$W(P)$	Bandwidth ratio of transmission-line band-stop filter; width of stop band at its low-attenuation edges (reflection coefficient $=P$) normalized to its center frequency
$W(S)$	Bandwidth ratio of band-stop filter; width of stop band at its high-attenuation edges [attenuation $=A(S)$] normalized to its center frequency

TABLE II

BAND-STOP FILTER WITH ALTERNATING QUARTER-WAVE AND HALF-WAVE STUBS AND QUARTER-WAVE LINES

$N=5$, Theta = 10 Degrees, $A(S)=83.9$ dB												$P=0.01$, Omega(S) = 5.759, Omega(1) = 1.0
$C(1)=0.4792$		$C(2)=0.0101$		$L(2)=1.0400$		$C(3)=1.2090$	$C(4)=0.0270$		$L(4)=1.0120$		$C(5)=0.4629$	
$W(P)$	$W(S)$	Z_1	Z_{12}	Z_{*2}	Z_{**2}	Z_{23}	Z_3	Z_{34}	Z_{*4}	Z_{**4}	Z_{45}	Z_5
0.60	0.112	6.153	1.194	2.032	743.570	1.336	1.604	1.331	2.103	296.526	1.185	6.402
0.70	0.135	5.465	1.224	1.721	435.458	1.413	1.332	1.407	1.786	174.102	1.213	5.695
0.80	0.160	4.935	1.254	1.482	266.780	1.501	1.121	1.494	1.543	107.009	1.241	5.151
0.90	0.187	4.509	1.285	1.291	168.109	1.603	0.953	1.595	1.349	67.711	1.269	4.717
1.00	0.219	4.156	1.317	1.132	107.547	1.723	0.812	1.714	1.190	43.551	1.298	4.358
1.10	0.255	3.856	1.350	0.997	69.074	1.868	0.693	1.858	1.055	28.170	1.327	4.056
1.20	0.299	3.595	1.385	0.878	44.058	2.046	0.589	2.035	0.939	18.142	1.358	3.797
1.30	0.352	3.364	1.423	0.773	27.581	2.274	0.496	2.263	0.837	11.511	1.389	3.571
1.40	0.418	3.156	1.464	0.677	16.702	2.577	0.412	2.565	0.748	7.109	1.421	3.376
1.50	0.505	2.969	1.508	0.589	9.594	3.003	0.334	2.991	0.670	4.209	1.453	3.210
1.60	0.625	2.800	1.556	0.506	5.076	3.645	0.262	3.634	0.606	2.341	1.481	3.080
1.70	0.797	2.652	1.605	0.431	2.359	4.727	0.193	4.719	0.563	1.189	1.497	3.014
1.80	1.058	2.545	1.647	0.372	0.886	6.919	0.127	6.922	0.577	0.530	1.467	3.139

$N=5$, Theta = 15 Degrees, $A(S)=66.1$ dB												$P=0.01$, Omega(S) = 3.864, Omega(1) = 1.0
$C(1)=0.4696$		$C(2)=0.0230$		$L(2)=1.0280$		$C(3)=1.1890$	$C(4)=0.0630$		$L(4)=0.9653$		$C(5)=0.4324$	
0.50	0.136	7.272	1.159	2.500	615.428	1.266	1.984	1.257	2.694	258.235	1.143	7.990
0.60	0.167	6.316	1.188	2.070	336.835	1.336	1.607	1.324	2.241	141.993	1.168	6.966
0.70	0.200	5.618	1.217	1.758	197.711	1.413	1.331	1.400	1.914	83.830	1.191	6.224
0.80	0.237	5.080	1.245	1.518	121.467	1.502	1.119	1.487	1.665	51.878	1.215	5.659
0.90	0.277	4.650	1.274	1.326	76.810	1.604	0.949	1.587	1.468	33.108	1.237	5.214
1.00	0.322	4.296	1.303	1.169	49.359	1.725	0.808	1.706	1.309	21.527	1.259	4.855
1.10	0.375	3.996	1.334	1.035	31.886	1.870	0.687	1.849	1.177	14.121	1.281	4.562
1.20	0.436	3.737	1.365	0.919	20.497	2.050	0.583	2.028	1.067	9.264	1.301	4.323
1.30	0.509	3.512	1.398	0.818	12.971	2.279	0.490	2.256	0.976	6.028	1.319	4.137
1.40	0.598	3.314	1.432	0.728	7.979	2.585	0.406	2.562	0.903	3.856	1.332	4.009
1.50	0.711	3.143	1.467	0.648	4.696	3.015	0.330	2.993	0.851	2.402	1.337	3.964
1.60	0.856	3.003	1.499	0.580	2.586	3.665	0.258	3.647	0.830	1.441	1.324	4.087
1.70	1.048	2.913	1.523	0.530	1.291	4.759	0.190	4.754	0.867	0.822	1.266	4.753
1.80	1.301	2.962	1.510	0.527	0.559	6.981	0.125	7.012	1.056	0.436	1.082	13.167

TABLE III
RANGE OF PARAMETERS IN THE DESIGN TABLES

P	$A(S)$	Range of $W(P)$		Range of $W(S)$	
		From	To	From	To
0.01	From 83.9 To 28.1	0.60 0.30	1.80 0.60	0.112 0.174	1.058 0.362
0.03	From 93.4 To 25.7	0.60 0.30	1.80 0.50	0.112 0.214	1.058 0.363
0.05	From 97.9 To 24.9	0.60 0.20	1.70 0.50	0.112 0.154	0.797 0.391
0.10	From 103.9 To 21.4	0.50 0.20	1.70 0.40	0.091 0.174	0.797 0.349
0.15	From 107.5 To 20.3	0.50 0.20	1.70 0.40	0.091 0.182	0.797 0.365
0.25	From 112.1 To 20.4	0.50 0.20	1.70 0.60	0.091 0.188	0.797 0.569
0.50	From 119.1 To 17.5	0.50 0.30	1.60 0.70	0.091 0.296	0.625 0.691

equal at two, or even three, frequencies. This technique appears to be suitable for filters with wide stop bands, for then the matching frequencies can be the design frequency and the edges of the stop band. The result will be a broadband match. When three matching frequencies are used, the impedance of the substitute stub will be fully determined, and it is then entirely possible that the substitute stub impedance will not be significantly lower than the exact stub impedance. Thus, nothing will have been gained by the substitution, and it will be necessary to relax the broadbanding procedure and match the stubs at only two frequencies. This latter technique was used in a trial filter to be described later. The described methods of stub substitution apply only to the quarter-wave stub and to the end section of the half-wave stub.

After the normalized stub and line impedances have been determined they are multiplied by the desired terminating impedances, and the filter is constructed by standard methods.

How the Tables Were Computed

Restricting ourselves to consideration of the case $N=5$, we find that the problem is to determine the impedances of each quarter-wave section of the TL filter shown in Fig. 1. The prototype with which we must work is the elliptic-function low-pass prototype of Fig. 3. The prototype filter has two inductors and five capacitors whose values are given in the Telefunken catalogue [9]. The filters in the catalog have the amplitude response pictured in Fig. 4, cutoff frequencies of $\Omega(1)=1.0$, and terminating impedances of 1.0 ohm. The impedance level of 1.0 ohm is maintained throughout this procedure; however, the cutoff frequency must be changed so that after the final conversion to a band-stop filter we arrive at the specified bandwidth $W(P)$ (see Fig. 2). This is most easily accomplished by assigning new element values to the filter in Fig. 3 as follows,

$$C_i = \Lambda C_i' \quad \text{and} \quad L_i = \Lambda L_i' \tag{2}$$

where the parameter Λ is defined by

$$\Lambda = \Omega(1) \tan \left[(\pi/4) W(P)\right] \tag{3}$$

and the primed values of C and L are those given in the catalog tables [9]. The effect of this conversion is to change every product (LC) by the factor Λ^2, while keeping invariant every quotient (L/C). Since the former has the dimension of frequency squared while the latter has the dimension of impedance squared, we see that the net effect is to maintain constant the impedance level of the filter and to multiply all frequencies by the factor Λ. It is important that this conversion of cutoff frequency be made at the outset. Next, comparing Figs. 1 and 3, we see that the former cannot be directly derived from the latter, because the configurations of these two filters are different. Figure 5, however, shows an elliptic-function LE low-pass filter that is suitable for conversion to a band-stop filter by direct transformation. Here, unit elements have been inserted between the adjacent branches, all of which are now in shunt. Note that the two parallel-resonant series branches have been converted to series-resonant shunt branches. The frequency of resonance in each case is a frequency of infinite attenuation indicated in Fig. 4.

Next, using the three transformations of Fig. 6 for converting lumped elements (L, C, and unit elements) to transmission-line sections, we obtain from Fig. 5 the final TL form of the band-stop filter shown in Fig. 1. The (element) transformations and formulas of Fig. 6(a) and 6(b) convert a capacitor to an open-circuited section of transmission line, and a unit element to a section of connecting line (unit line). These conversions are derived from Richards' tangent transformation [1]

$$\Omega = \tan \left[(\pi/2)(\omega/\omega_0)\right]. \tag{4}$$

Transformation equation (4) maps the band-stop filter frequency domain ω into that of the low-pass filter Ω. Here, ω_0 is the design frequency (center of the stop band). It is assumed, of course, that the prototype filter has first been converted according to (2) so that its element values reflect the altered cutoff frequency which, after making the changeover from lumped elements to transmission-line elements, yields the desired width of stop band. The element transformation and formulas of Fig. 6(c) also are derived from the tangent transformation equation (4). In addition, they represent a conversion from a series network to a cascade network.

Up to this point all steps pertinent to the computation of the elements of the TL filter have been outlined ex-

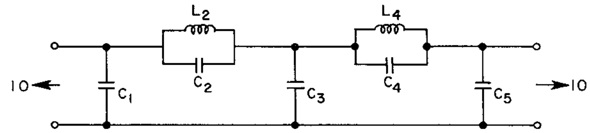


Fig. 3. Prototype lumped-element filter on which the transmission-line filter of Fig. 1 is based.

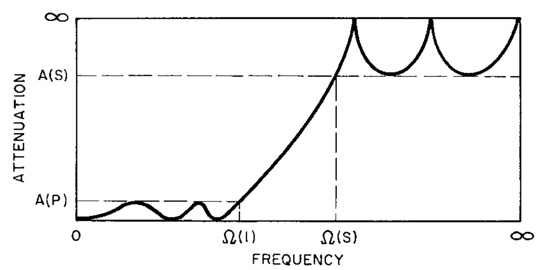


Fig. 4. Response of the filter of Fig. 3.

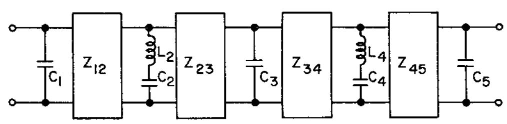


Fig. 5. Intermediate form of the lumped-element prototype with unit elements separating reactive branches.

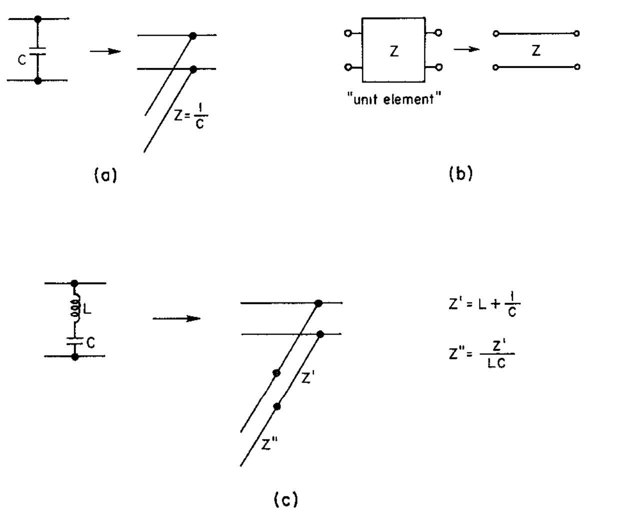


Fig. 6. Transformations from lumped elements to transmission-line sections. (a) Capacitor to open-circuit quarter-wave stub. (b) Unit element to quarter-wave connecting line. (c) Series-resonant circuit to half-wave open-circuit stub.

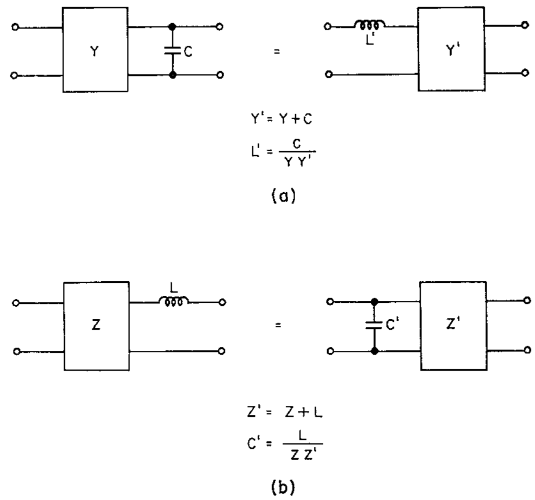


Fig. 7. Kuroda's identities.

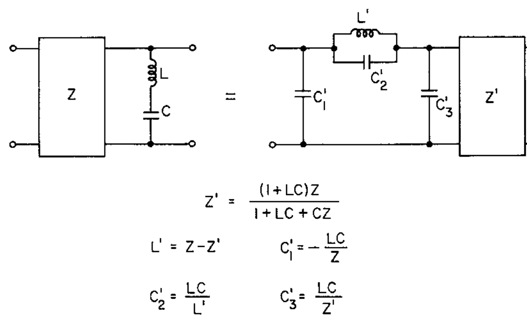


Fig. 8. Kuroda-Levy identity.

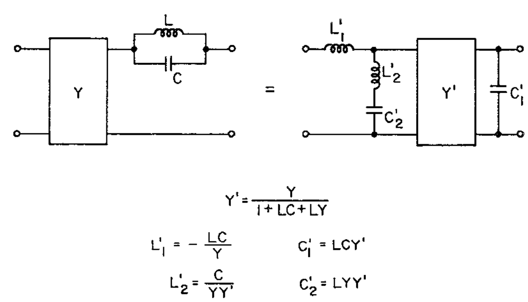


Fig. 9. A modification of the Kuroda-Levy identity more suitable for computer programming than the form of Fig. 8.

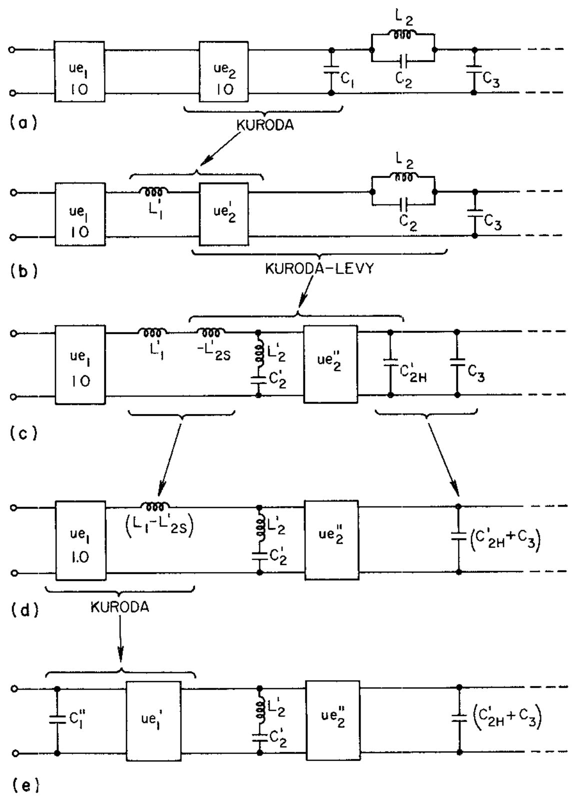


Fig. 10. Steps used in converting the lumped-element filter of Fig. 3 to that of Fig. 5.

cept the method of converting Fig. 3 into Fig. 5. For this conversion, which represents the bulk of the total operation, we use the Kuroda and the Kuroda-Levy identities, applied sequentially. Kuroda's identities are given in Fig. 7, and the Kuroda-Levy identity in Fig. 8. The basic idea is to work unit elements into the filter network from one end. Since each unit element here has an impedance of 1 ohm (the terminating impedance) before insertion, it does not change the amplitude response of the filter. However, each unit element serves the purpose of separating adjacent reactive branches and converting them into shunt branches. Kuroda's identities are applied to the one-element series and shunt branches of the LE filters while the Kuroda-Levy identity is applied to the two-element branches. Figure 9 shows a slightly modified form of the Kuroda-Levy formula which was found to be more directly applicable to the computer program than that of Fig. 8.

The various transformations are applied according to the scheme of Fig. 10. Briefly, the network of Fig. 3 is first divided into two halves and each half is treated separately (as in Fig. 10); the halves are then recombined to form the network of Fig. 5. Note that only one of the halves contains the capacitor C_3. In the conversion process, two unit elements are first added on the terminal end of each half, as shown in Fig. 10(a). The sequence of operations involving the identities is then followed as given in steps (a) through (e). Note that in the transition from (c) to (d) a negative-valued (nonrealizable) inductor is combined with a positive inductor to yield either a positive or negative value of inductance. When the result is positive, the filter is realizable; otherwise the filter is not realizable in the desired form. The latter condition was always found to occur in the upper portion of the range of the normalized value of the stop-band width $W(P)$. Consequently, the bandwidth parameter $W(P)$ of the design tables is limited on the high side, particularly for low values of $A(S)$. The upper limit on the characteristic impedance of any section of a filter was arbitrarily chosen as having a normalized value of 1000 with relation to the terminating impedance. Thus the lower range of $W(P)$ was omitted leaving only the medium range of that parameter.

Trial Filter

To test the computed filter designs, a filter having the following parameters was chosen:[5]

maximum reflection coefficient in pass band

$$P = 0.25 \quad (\text{VSWR} = 1.67)$$

minimum attenuation in stop band

$$A(S) = 41.2 \text{ dB}$$

fractional bandwidth of stop band between edges of first and second pass bands

$$W(P) = 1.0$$

fractional bandwidth of stop band at minimum attenuation points

$$W(S) = 0.814.$$

The characteristic impedance of each quarter-wave section of the filter normalized to the terminating impedance, and corresponding values of impedance for a 50-ohm system are given in Table IV.[5]

TABLE IV

Impedances of Elements of Trial Band-Stop Filter

Element	Normalized to Z_0	Ohms in a 50-Ohm System
Z_1	3.195	159.7
Z_{12}	1.456	72.8
Z_2'	0.810	40.5
Z_2''	3.337	166.9
Z_{23}	1.659	82.9
Z_3	0.445	22.25
Z_{34}	1.565	78.3
Z_4'	1.473	73.7
Z_4''	2.853	142.7
Z_{45}	1.241	62.1
Z_5	5.147	257.3

[5] The trial filter design was chosen from an earlier and slightly different compilation of design parameters and is not recorded in the tables.

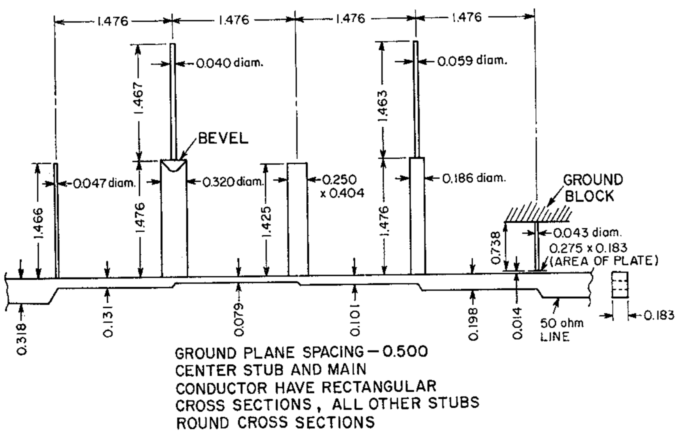


Fig. 11. Detail drawing of the stubs and center conductor of the trial filter.

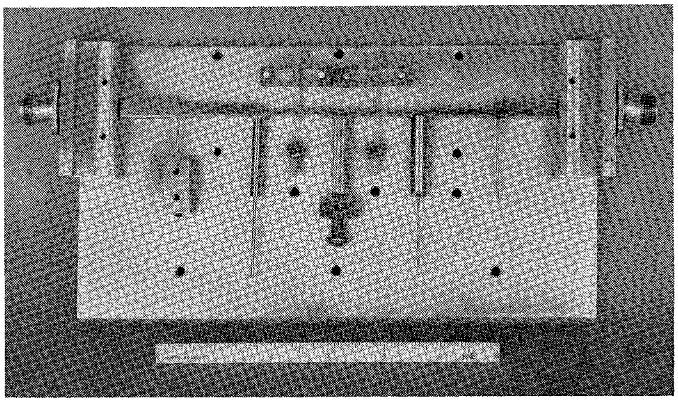


Fig. 12. Elliptic-function trial filter with cover plate removed.

Realizing the Trial Filter in Strip Line

The center frequency of the trial filter was chosen to be 2.0 GHz. The first pass band is thus 0 to 1.0 GHz, and the first stop band is 1.0 to 3.0 GHz. The second pass band extends from 3.0 to 5.0 GHz. A quarter wave at the design frequency 2.0 GHz is 1.476 inches; therefore, in order to have low circuit losses and reasonably small discontinuity and end effects, the ground-plane spacing was made ½ inch, with air as the dielectric. All connecting lines and resonators (stubs) except Stub 5 ($Z_5 = 257.3$ ohms) were found to be directly realizable. On the other hand, Stub 5 would have been very slender and fragile, even with ½ inch spacing of the ground planes, if it were realized exactly; therefore a shorter, but thicker, capacitively coupled short-circuited stub was used instead.

The dimensions of the stubs and lines are shown in Fig. 11. All stubs except the middle one have circular cross sections [13], and the connecting lines and the center stub have rectangular cross sections [14]. The length of each open-circuited stub was reduced a small amount to allow for the effect of end capacity. Also, Stub 2 was beveled at the junction of the low and high impedance sections to reduce the detuning effect of the discontinuity capacitance. The center stub has the lowest impedance (40.5 ohms) and is one quarter-wave long. Thus it accounts for most of the attenuation at the center of the stop band, and also has a strong effect on the overall filter performance. Therefore it had to be precisely tuned. The center stub was tuned by a screw at its end, as shown in the internal parts of the filter (Fig. 12). (This last detail has been omitted from Fig. 11.) The capacitor at the junction of Stub 5 with the main line consisted of a small metal tab soldered to the stub and separated from the line by an appropriate amount.

Design of the Capacitively-Coupled Stub (No. 5)

The capacitively-coupled stub design is based on the requirement that the input impedance of the exact and substitute stubs be equal at the lower edge and at the center of the stop band. This two-point matching scheme results in a substitute stub input impedance that matches the exact stub input impedance (Z_5) over a very wide band. The excellent fit is evidenced in the graph (Fig. 13) which shows the impedance Z_{in} vs. frequency of a high-impedance stub of normalized characteristic impedance $Z_0 = 5$ ohms[6] (solid line) compared with the input impedance of the capacitively-coupled stub designed for a two-point match, as described above (short-dashed line). Note that in Fig. 13 the electrical length θ of the exact design stub (one quarter-wave long at the center of the stop band) has been substituted for the frequency variable, so that $\theta = 90°$ in that figure represents the center of the stop band.

A second method of matching was also investigated, as shown by the long-short-dashed curve of Fig. 13. In this case it was required that both the input impedance and the reactance slope of the two stubs be matched at the center of the stop band [11], [12]. This method is seen to be unsuited to the present wide-band case. The electrical length of the capacitively-coupled stub was chosen as 45 degrees, since this value both yielded a substantial reduction in the stub impedance (162.5 ohms as against $Z_5 = 257.3$ ohms) and provided a broadband match, as indicated in Fig. 13. (A trial value of 60 degrees for the shortened stub length, also using a two-point match, was found to yield a still lower stub impedance. The input impedance of the capacitively-coupled stub, however, did not then match that of the quarter-wave stub over as wide a band as did the 45-degree stub.)

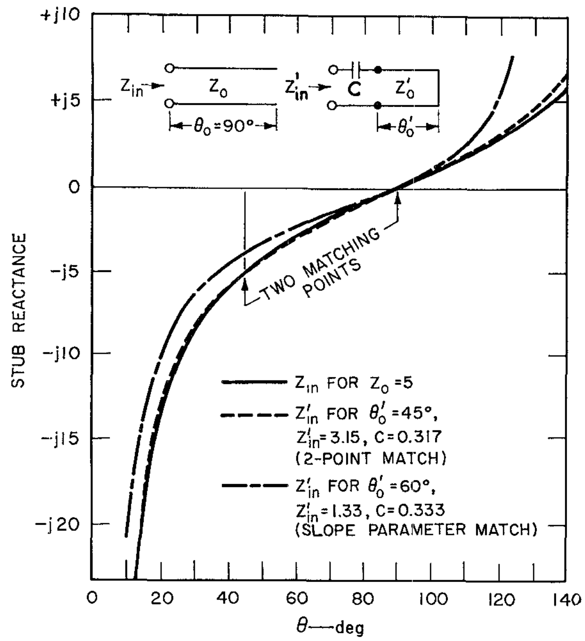
The calculations for the capacitively-coupled shorted stub proceeded from the following expressions for stub input impedance. For the exact stub

$$Z_{in} = -Z_0 \cot \theta, \qquad (5)$$

and for the substitute stub

$$Z_{in}' = Z_0' \tan \theta' - \frac{1}{\omega C}, \qquad (6)$$

[6] Here the normalized stub impedance is slightly smaller than that of Stub 5 of the trial filter; however, the same result holds in the latter case as well.



NOTE: All numerical values are normalized to a terminating impedance of 1 ohm and a radian frequency of 1 rad/s at midband

Fig. 13. Comparison of exact and approximate stub reactances.

where θ is the electrical length of a quarter-wave stub at radian frequency ω, θ' is the electrical length of the shortened stub at the same frequency, C is the series capacitance required to tune the shortened stub at the design frequency ω_0, and Z_0 and Z_0' are the characteristic impedances of the exact quarter-wave stub and the shortened stub, respectively. Note that $\theta_0 = \pi/2$ is the electrical length of the exact stub at $\omega = \omega_0$. The expressions for Z_{in} and Z_{in}' were equated at the design frequency ($\theta = \theta_0$) and at the lower edge of the stop band ($\theta = \pi/4$), and were then solved for the two unknowns, C and Z_0'.

In this case the calculated values are $C = 0.308/\omega_0$ and $Z_0' = 3.25$ ohms, both normalized to the terminating impedance. These values are then converted to an actual capacitance of $C = 0.49$ pF and 162.5 ohms. The calculated capacitance was used as a guide in tuning the shortened stub to resonate at ω_0. Resonance was indicated by an attenuation peak at ω_0, with the other quarter-wave stubs shorted at their ends.

Measured Performance of Trial Filter

The measured attenuation of the trial filter is shown in Fig. 14. In general the response shape of the filter is in good conformity with the theoretical response, as indicated by the delineations of pass bands and stop bands and by the dashed lines indicating the stop band at the high attenuation point, 41.2 dB. VSWR measurements in the range 0 to 5.0 GHz (excluding the stop band) are shown in Fig. 15. These indicate very good conformance with theory in the first pass band (note the close agreement between measured and theoretical VSWR in that region). The need to resort to a capacitively-coupled

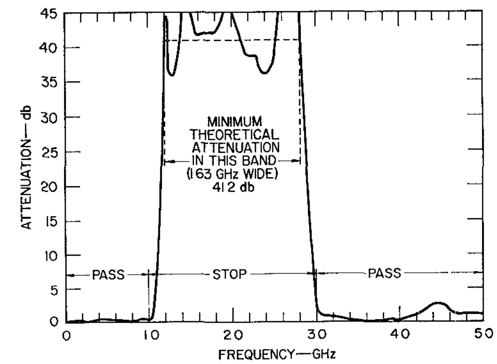


Fig. 14. Measured attenuation of the filter of Fig. 12.

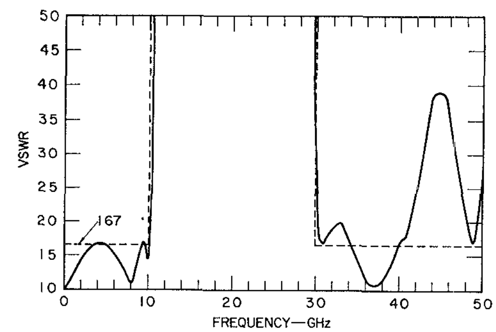


Fig. 15. Measured VSWR of the filter of Fig. 12.

stub is believed to have had relatively little adverse effect on the response in the stop band, or in the first pass band. The deterioration of the VSWR in the second pass band may have been caused by junction discontinuities, and the loss of full theoretical attenuation in the stop band is most likely attributable to that cause as well.

Conclusion

Because of the sharply rising attenuation skirts with relatively few elements, the elliptic-function microwave band-stop filter appears quite attractive. There are, however, severe restrictions on usable bandwidths for the ($N = 5$) case which apply to the open-circuited shunt-stub type of filter described herein.

Acknowledgment

A special note of thanks is due to Dr. R. Levy of the University of Leeds, Yorkshire, England, who made available to us, before publication, his circuit theory work without which the computation of the design tables would not have been undertaken.

We are also grateful to Mrs. Barbara Wheeler of the Computer Techniques Laboratory and to Miss Phyllis Groll of the Mathematical Sciences Department for excellent jobs of programming the filter analysis and synthesis, respectively, on the Burroughs B5500 Computer at the Stanford Research Institute and to W. Cornelius for an excellent job of both constructing and electrically testing the trial filter.

REFERENCES

[1] P. I. Richards, "Resistor-transmission-line circuits," *Proc. IRE*, vol. 36, pp. 217–220, February 1948.
[2] H. Ozaki and J. Ishii, "Synthesis of a class of strip-line filters," *IRE Trans. on Circuit Theory*, vol. CT-5, pp. 104–109, June 1958.
[3] A. I. Grayzel, "A synthesis procedure for transmission line networks," *IRE Trans. on Circuit Theory*, vol. CT-5, pp. 172–181, September 1958.
[4] B. M. Schiffman and G. L. Matthaei, "Exact design of band-stop microwave filters," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 6–15, January 1964.
[5] E. G. Cristal, G. L. Matthaei, and D. B. Weller, "Novel microwave filter design techniques," Stanford Research Institute, Menlo Park, Calif., 3rd Quarterly Progress Rept., Contract DA 36-039-AMC-00084(E), SRI Project 4344, pp. 3–29, September 1963. See also, E. G. Cristal, "Addendum to an exact method for synthesis of microwave band-stop filters," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 369–382, May 1964.
[6] H. J. Riblet, "The application of a new class of equal-ripple functions to some familiar transmission-line problems," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 415–421, July 1964.
[7] R. J. Wenzel, "Exact design of TEM networks using quarter-wave lines," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 94–111, January 1964.
[8] R. Saal and E. Ulbrich, "On the design of filters by synthesis," *IRE Trans. on Circuit Theory*, vol. CT-5, pp. 284–327, December 1958.
[9] R. Saal, "The design of filters using the catalogue of normalized low-pass filters," *Telefunken G.M.B.H.*, Backnang/Württ, Western Germany, 1963.
[10] R. Levy, "A generalized equivalent circuit transformation for distributed networks," *IEEE Trans. on Circuit Theory* (*Correspondence*), vol. CT-12, pp. 457–458, September 1965.
[11] L. Young, G. L. Matthaei, and E. M. T. Jones, "Microwave band-stop filters with narrow stop bands," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-10, pp. 416–427, November 1962.
[12] B. M. Schiffman, "Capacitively-coupled stub filter," *IEEE Trans. on Microwave Theory and Techniques* (*Correspondence*), vol. MTT-13, pp. 253–254, March 1965.
[13] R. M. Chisholm, "The characteristic impedance of trough and slab lines," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-4, pp. 166–172, July 1956.
[14] R. H. Bates, "The characteristic impedance of shielded slab lines," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-4, pp. 28–33, January 1956.
[15] J. Ishii, "Synthesis of commensurable transmission-line composite filters," Kinki University, Fuse City, Japan. See also K. Kuroda, "Derivation methods of distributed constant filters from lumped constant filters," 1952; this paper refers to a text for lectures at Joint Meetings of Kansai Branch of Three Institutes.

Propagation Characteristics of a Partially Filled Cylindrical Waveguide for Light Beam Modulation

D. CHEN, MEMBER, IEEE, AND T. C. LEE

***Abstract*—Propagation characteristics of a cylindrical waveguide partially filled with a cylindrical dielectric light modulation material is analyzed and numerical computation is performed for a few typical cases. This type of traveling-wave structure supports a TE or TM mode, and therefore is useful for light beam modulation applications requiring a longitudinal magnetic or electric field. Numerical analysis indicates that for the TM case, a broadband region occurs near a crossover point and broadband synchronization between the modulating microwave and the parallel-launched light beam can be obtained by a suitable choice of the dielectric medium surrounding the central dielectric.**

INTRODUCTION

BROADBAND microwave modulation of a coherent light beam calls for a microwave traveling-wave structure which provides 1) a broad nondispersive region in the ω-β relation and 2) matching between the phase velocity of the microwave signal and the component velocity of light along the microwave traveling direction [1], [2]. To satisfy the first requirement one of the approaches is to utilize a microwave circuit possessing TEM wave-like propagating properties. To satisfy the second requirement, it is generally necessary to zigzag the optical path along the waveguide since for most electrooptic or magnetooptic materials the index of refraction is much less than the square root of the dielectric constant at the modulation frequency. Thus far, only one simple structure has been proposed [2] in which a dielectric material is partially filled between two parallel plates, and such a circuit resembles a TEM waveguide when the dielectric constant of the electrooptic material approaches that of the surrounding dielectric medium. It is this feature of partial filling that provides two distinct nondispersive regions in the ω-β diagram as analyzed by Kaminow and Liu [2]. Transverse electric fields are employed for modulation in their circuit. Based on this circuit DiDomenico and Anderson have discussed extensively the various features that influence the modulation performance [3].

Whereas this type of TEM waveguide is simple and useful, it is, however, desirable in many instances to modulate a light beam by a longitudinal field. Modulation by the magnetooptic Faraday effects for example, requires such a field. It is therefore important to study microwave propagation circuits supporting a TE or TM mode. In this paper, we shall investigate the propagation characteiistics of a cylindrical waveguide partially

Manuscript received November 24, 1965; revised June 17, 1966.
The authors are with the Honeywell Corporate Research Center, Hopkins, Minn.